Int. J. Heat Mass Transfer. Vol. 24, No. 9, pp. 1531 1539, 1981.
Printed in Great Britain

0017 9310/81/091531 09 $02.00/0


# UNSTEADY THERMAL ENTRANCE HEAT TRANSFER OF POWER-LAW FLUIDS IN PIPES AND PLATE SLITS

HSIAO-TSUNG LIN and YEN-PING SHIH

Department of Chemical Engineering, National Cheng Kung University, Tainan, Taiwan, China

(*Received* 29 *October* 1980 *and in revised form* 25 *March* 1981)

**Abstract**—An instant-local similarity method is proposed to analyze the unsteady state Graetz problems. Unsteady heat transfer for fully-developed laminar flow of power-law non-Newtonian fluids in the thermal entrance region of pipes and plate slits, with viscous dissipation considered, is studied. For the unsteady thermal entrance heat transfer problems, only large Graetz numbers (small normalized axial distance) are concerned and the normalized time from transient up to steady state is of order $10^{-1}$; therefore, the instant-local similarity approach gives results of high accuracy. The effects of the flow index, viscous dissipation and Graetz number on the heat transfer rate are demonstrated with numerical solutions. The corresponding steady-state Graetz problems are studied by the local similarity method whose solutions agree very well with the extended Leveque solutions particularly for large Graetz number and Brinkman number.

## NOMENCLATURE

- $Br$, Brinkman number;
- $c_p$, specific heat;
- $Gz$, Graetz number, $=Q/(\alpha x)$ in pipe case and $=2HQ/(\alpha w x)$ in plate slit case;
- $h$, heat transfer coefficient;
- $H$, plate slit half-height;
- $k$, thermal conductivity;
- $K$, parameter of a power-law fluid;
- $m$, $1/n$;
- $n$, flow index of a power-law fluid;
- $Nu$, local Nusselt number, $=2hR/k$ in pipe case and $=2hH/k$ in plate slit case;
- $Q$, volumetric flow rate, $=\pi R^2\langle u\rangle$ in pipe case or $2wH\langle u\rangle$ in pipe slit case;
- $r$, radial coordinate in pipe;
- $R$, pipe radius;
- $t$, time;
- $T$, temperature of the fluid;
- $T_b$, bulk temperature;
- $T_0$, inlet fluid temperature;
- $T_s$, inside surface temperature;
- $u$, velocity in axial coordinate;
- $u_{max}$, maximum velocity in the axial direction, $=\langle u\rangle\ (m+3)/(m+1)$ for pipe flow and $=\langle u\rangle\ (m+2)/(m+1)$ for plate slit flow;
- $\langle u\rangle$, average velocity in the axial direction;
- $w$, width of the plate slit;
- $x$, axial coordinate;
- $X$, normalized axial coordinate, $=\alpha x/(R^2 u_{max})$ in pipe case and $=\alpha x/(H^2 u_{max})$ in plate slit case;
- $y$, cross-slit coordinate;
- $Y$, normalized radial coordinate, $=r/R$; normalized cross slit coordinate, $=y/H$;
- $1(t)$, Heaviside unit operator, $=0$ for $t<0$ and $=1$ for $t\geq 0$.

Greek symbols

- $\alpha$, thermal diffusivity;
- $\zeta$, $\eta\xi\tau^{-1/2}$;
- $\eta$, transformed radial or cross-slit coordinate, $=(1-Y)/\xi$;
- $\Theta$, normalized temperature, $=(T-T_s)/(T_0-T_s)$;
- $\tau$, normalized time, $=\alpha t/R^2$ for pipe flow and $=\alpha t/H^2$ for plate slit flow;
- $\xi$, transformed axial coordinate, $=(9X/2)^{1/3}$;
- $\rho$, density.

## 1. INTRODUCTION

THE CLASSICAL steady-state heat and mass transfer in the entrance region of channels with fully-developed laminar flow is well known as the Graetz problem. The eigenfunction expansion method is used extensively for the study of this problem. Recently, it is shown that the extended Leveque method [1–6] is useful for large Graetz number which corresponds to small normalized axial distance. Therefore, the extended Leveque solutions are the supplemental solutions of the eigenfunction solutions which require less eigenfunctions for small Graetz number.

The unsteady Graetz problems were analyzed by Sparrow and Siegel [7, 8] using the methods of characteristics and finite difference. Siegel [9] treated the same problem in the downstream region using eigenfunction expansion method. Only Newtonian fluids without viscous dissipation are considered in the previous analyses. However, in the design of the control systems of heat transfer devices in organic-cooled nuclear reactors, non-Newtonian fluids are concerned with and viscous dissipation is significant.

In this paper, the unsteady state Graetz problems for the heat transfer in the thermal entrance region for fully-developed laminar flow of power-law non-

Newtonian fluids in pipes and plate slits with step change in surface temperature are studied by a new method called instant-local similarity method proposed by the authors. The new method uses the concept of the extended Leveque method by restricting the solutions to large Graetz number and converting the energy equations to boundary layer type. The comparison of the local similarity solution with the extended Leveque solution for the steady Graetz problems in pipes is given first.

## 2. GOVERNING EQUATIONS

The unsteady-state energy equation in the thermal entrance region for fully-developed laminar flow in a pipe of power-law fluids with constant physical properties is described by the following equation

$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p u \frac{\partial T}{\partial x} = \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + K\left|\frac{du}{dr}\right|^{n-1}\left(\frac{du}{dr}\right)^2 \tag{1}$$

where

$$u = u_{max}\left[1-\left(\frac{r}{R}\right)^{m+1}\right], \quad m = \frac{1}{n}. \tag{2}$$

The last term of equation (1) denotes viscous dissipation, and $n$ is the flow index of the power-law non-Newtonian fluids. Conduction and dispersion in the axial direction have been neglected. The initial and boundary conditions are

$$T(x, r, 0) = T_0 \tag{3}$$

$$T(0, r, t) = T_0 \tag{4}$$

$$\frac{\partial T}{\partial r}(x, 0, t) = 0 \tag{5}$$

$$T(x, R, t) = T_0 + (T_s - T_0)\,1(t) \tag{6}$$

where $1(t)$ is the Heaviside unit operator,

$$1(t) = \begin{cases} 1, t \geq 0 \\ 0, t < 0. \end{cases} \tag{7}$$

Equations (3) and (4) show that the initial temperature of the fluid in the pipe and the entrance fluid temperature are both at a constant temperature $T_0$. Equation (6) denotes that the pipe wall has a constant wall temperature $T_s$ for time $t \geq 0$.

Defining the following dimensionless variables and groups

$$Y = \frac{r}{R}$$

$$X = \frac{\alpha x}{R^2 u_{max}}$$

$$\tau = \frac{\alpha t}{R^2} \tag{8}$$

$$\Theta = \frac{T - T_s}{T_0 - T_s}$$

$$Br = (m+1)^{n+1}\frac{KR^2}{k(T_0 - T_s)}\left(\frac{u_{max}}{R}\right)^{n+1}$$

equations (1)–(6) become

$$\frac{\partial \Theta}{\partial \tau} + (1 - Y^{m+1})\frac{\partial \Theta}{\partial X} = \frac{1}{Y}\frac{\partial}{\partial Y}\left(Y\frac{\partial \Theta}{\partial Y}\right) + Br\,Y^{m+1} \tag{9}$$

$$\Theta(X, Y, 0) = 1 \tag{10}$$

$$\Theta(0, Y, \tau) = 1 \tag{11}$$

$$\frac{\partial \Theta}{\partial Y}(X, 0, \tau) = 0 \tag{12}$$

$$\Theta(X, 1, \tau) = 0. \tag{13}$$

Equations (9)–(13) form a linear partial differential equation system whose exact solution is very difficult to obtain. Therefore, we are trying to find the approximate solutions for various limiting cases. One of the limiting cases is the steady state solution for large Graetz number. Leveque solution [10] which assumes a linear velocity profile with respect to normal distance from the pipe wall is well known. The Leveque solution has been extended by a number of authors [1–6]. Basically, in the extended Leveque method the energy equation system is converted to a boundary-layer type by replacing the boundary condition at the center of the pipe with the boundary condition outside the thermal boundary layer. Therefore, local similarity method for the boundary layer heat transfer [11–14] can be applied to obtain a solution for large Graetz number. It is expected that the extended Leveque solution agrees with the local similarity solution. For the unsteady-state problem, an instant-local similarity method is proposed to find the approximate solution for small normalized time and large Graetz number. First, a coordinate transformation is used to change the independent variables $(X, Y, \tau)$ of $\Theta$ into $(\xi, \eta, \tau)$. That is

$$\left.\begin{matrix} X \\ Y \\ \tau \end{matrix}\right\} \Rightarrow \begin{cases} \xi = \left(\dfrac{9X}{2}\right)^{1/3} \\ \eta = \dfrac{1-Y}{\xi} \\ \tau \end{cases} \tag{14}$$

where $\xi$ is the transformed axial distance and $\eta$ is the transformed normal distance from the wall. Applying equations (14), equations (9), (10) and (13) become, respectively

$$\frac{\partial^2 \Theta}{\partial \eta^2} - \{(1-\xi\eta)^{-1}\xi - \frac{3}{2}\xi^{-1}[1-(1-\xi\eta)^{m+1}]\eta\}\frac{\partial \Theta}{\partial \eta} + Br\xi^2(1-\xi\eta)^{m+1} - \xi^2\frac{\partial \Theta}{\partial \tau} = \frac{3}{2}[1-(1-\xi\eta)^{m+1}]\frac{\partial \Theta}{\partial \xi} \tag{15}$$

$$\Theta(\xi, \eta, 0) = 1 \tag{16}$$

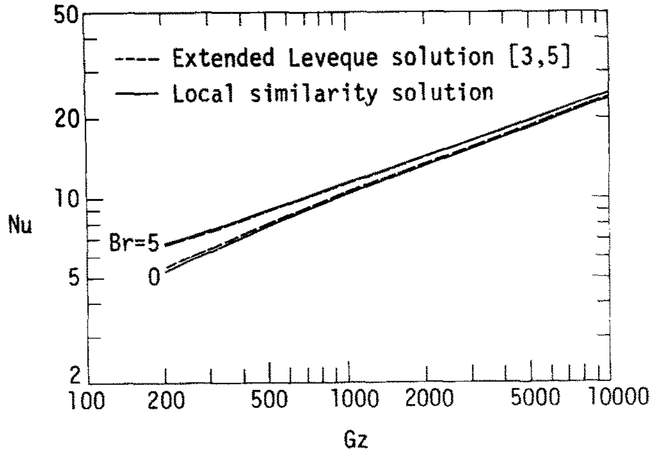


FIG. 1. Comparison of steady-state heat transfer for pipe flow, $n = 1$.

$$\Theta(\xi, 0, \tau) = 0. \tag{17}$$

Since we are trying to find approximate solutions in the thermal entrance region for large Graetz number, the heat transfer problem becomes boundary-layer type. Instead of using the boundary condition at the center of the pipe, equation (12), the boundary condition at the inlet of the pipe, equation (11), is used for the boundary condition outside the thermal boundary layer. That is

$$\Theta(\xi, \infty, \tau) = 1. \tag{18}$$

## 3. STEADY-STATE CASE

For the steady-state case, equations (15), (17) and (18) become

$$\frac{\partial^2\Theta}{\partial\eta^2} - \{(1-\xi\eta)^{-1}\xi - \frac{3}{2}\xi^{-1}[1-(1-\xi\eta)^{m+1}]\eta\}\frac{\partial\Theta}{\partial\eta} + Br\xi^2(1-\xi\eta)^{m+1} = \frac{3}{2}[1-(1-\xi\eta)^{m+1}]\frac{\partial\Theta}{\partial\xi} \tag{19}$$

$$\Theta(\xi, 0) = 0 \tag{20}$$

$$\Theta(\xi, \infty) = 1. \tag{21}$$

This is a boundary-layer heat transfer problem. The local similarity method [11–14] can be applied to obtain the approximate solution. The method is the deleting of the term containing $\partial\Theta/\partial\xi$ on the right-hand side of equation (19) and considering $\xi$ as a prescribed parameter. Notice that when $\xi$ is small, the term on the right-hand side of (19) is also small and can be omitted. The simplified equation from (19) is an ordinary differential equation which forms a two-point boundary-value problem with equations (20) and (21) as boundary conditions. Numerical solutions are obtained by a fixed step-size fourth-order Runge–Kutta–Gill integration scheme along with a conventional shooting method [15]. A description of the numerical method was given in [16].

The heat transfer coefficient $h$ is defined as

$$h(T_s - T_b) = k\left.\frac{\partial T}{\partial r}\right|_{r=R} \tag{22}$$

where $T_b$ is the bulk temperature. Since the local similarity solution is only applied to small axial distance, $T_b$ can be represented by the inlet temperature $T_0$. Hence the local Nusselt number, $Nu$, becomes

$$Nu = \frac{2hR}{k} = 2\xi^{-1}\left.\frac{\partial\Theta}{\partial\eta}\right|_{\eta=0}. \tag{23}$$

The Graetz number, $Gz$, is defined as

$$Gz = \frac{Q}{\alpha x} \tag{24}$$

where $Q$ is the volumetric flow rate. Thus

$$Gz = \frac{9\pi}{2}\left(\frac{m+1}{m+3}\right)\xi^{-3}. \tag{25}$$

Therefore, normalized axial distance is inverse proportional to the 1/3 power of the Graetz number.

Figure 1 shows the comparison of the local similarity solution with the extended Leveque solution [3, 5]. The agreement between these solutions is very good particularly for large Graetz numbers. The matching is excellent for large Brinkman number which corresponds to significant viscous dissipation. It is thus concluded that both the local similarity solution and the extended Leveque solution are valid for large Graetz number. Typical temperature profiles near the thermal inlet of a pipe are shown in Fig. 2 for $n = 1$ and $Br = 0$. As expected, the thickness of thermal boundary layer increases with a decrease of the Graetz number. In the next section, the idea of local similarity is extended to unsteady-state heat transfer.

## 4. THE INSTANT-LOCAL SIMILARITY METHOD

Deleting the term on the right-hand side of equation (15) we have the local similarity approximation of the unsteady-state heat transfer for small normalized

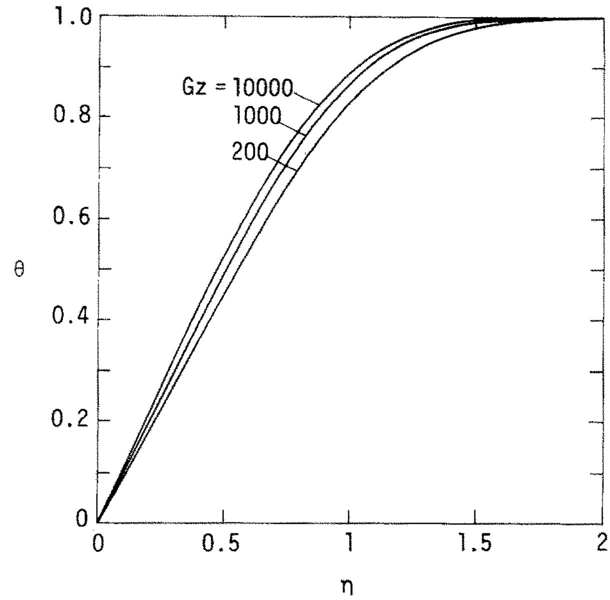


FIG. 2. Typical temperature profiles near the thermal entrance of a pipe, $n = 1$, $Br = 0$.

axial distance. In order to find the unsteady heat transfer solution, a further coordinate transformed from $(\xi, \eta, \tau)$ to $(\xi, \zeta, \tau)$ is made by defining

$$\zeta = (R-r)/(\alpha t)^{1/2} = \eta\xi\tau^{-1/2}. \tag{26}$$

Hence, equation (15), with the term on the right-hand side deleted by the local similarity approximation, becomes

$$\frac{\partial^2\Theta}{\partial\zeta^2} - \{(1-\tau^{1/2}\zeta)^{-1}\tau^{1/2} - \frac{3}{2}\xi^{-3}[1-(1-\tau^{1/2}\zeta)^{m+1}] \times \tau\zeta - \frac{1}{2}\zeta\}\frac{\partial\Theta}{\partial\zeta} + Br\tau(1-\tau^{1/2}\zeta)^{m+1} = \tau\frac{\partial\Theta}{\partial\tau}. \tag{27}$$

The transformed boundary conditions are

$$\Theta(\xi, 0, \tau) = 0 \tag{28}$$

$$\Theta(\xi, \infty, \tau) = 1. \tag{29}$$

For small $\tau$ and (or) small $\partial\Theta/\partial\tau$, the term $\tau(\partial\Theta/\partial\tau)$ on the right-hand side of equation (27) can be omitted, in a similar manner like the local similarity method, to simplify the partial differential equation into an ordinary differential equation. This is called the instant similarity method proposed by the authors. The successive use of the local similarity method and the instant similarity method is thus called the instant-local similarity method. Numerical solution is obtained by considering $\xi$ and $\tau$ as prescribed parameters in the numerical integration of the simplified equation

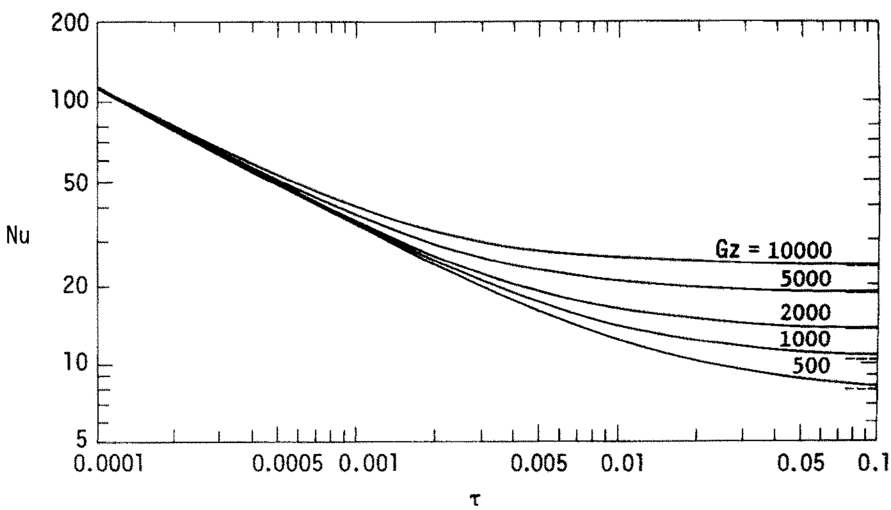


FIG. 3. Effect of Graetz number on transient heat transfer of pipe, $n = 1$, $Br = 0$. --- designates steady-state value.

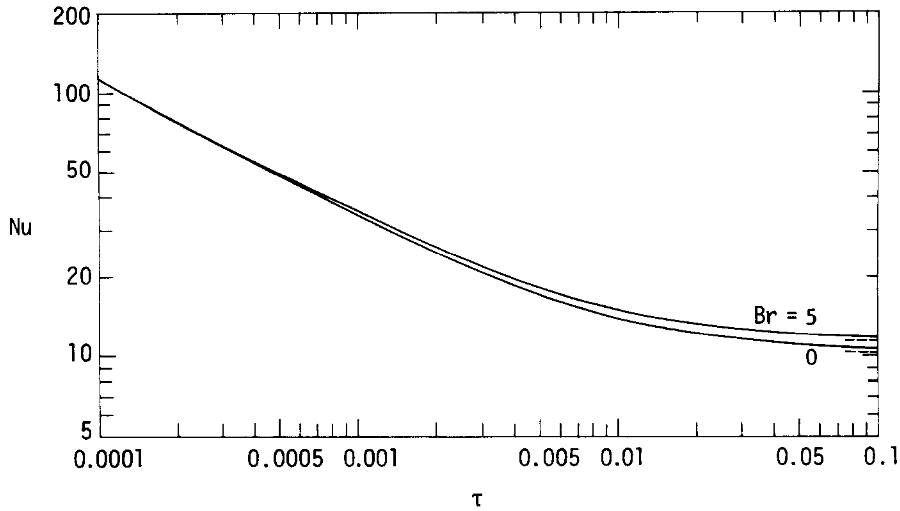


FIG. 4. Effect of Brinkman number on transient heat transfer of pipe, $n = 1$, $Gz = 1000$. --- designates steady-state value.

system by the numerical method mentioned above.

Expressing the heat transfer rate in terms of local Nusselt number

$$Nu = \frac{2hR}{k} = 2\tau^{-1/2}\frac{\partial\Theta}{\partial\zeta}(\xi, 0, \tau). \tag{30}$$

The effects of Graetz number, Brinkman number and flow index on the local Nusselt number are shown in Figs. 3–5, respectively. It is illustrated that the unsteady-state solutions approach to the corresponding steady-state values asymptotically. The steady-state values are obtained from extended Leveque solutions in [4, 5]. Development of temperature profile following a step change in surface temperature of pipe is shown in Fig. 6. The profile approaches to the steady state layer in a small normalized time.

## 5. PLATE SLIT CASE

The fully-developed steady flow of power-law non-Newtonian fluids in a plate slit between two parallel plates separate of height $2H$ is described by the velocity profile

$$u = u_{max}\left[1-\left(\frac{y}{H}\right)^{m+1}\right] \tag{31}$$

where $u$ is the velocity in the axial coordinate $x$, and $y$ is the normal coordinate from the center of the plate slit. The unsteady-state heat transfer equation of power-law fluids with constant physical properties and neglecting the heat conduction in the axial coordinate is

$$\rho c_p\frac{\partial T}{\partial t} + \rho c_p u\frac{\partial T}{\partial x} = k\frac{\partial^2 T}{\partial y^2} + K\left|\frac{du}{dy}\right|^{n-1}\left(\frac{du}{dy}\right)^2 \tag{32}$$

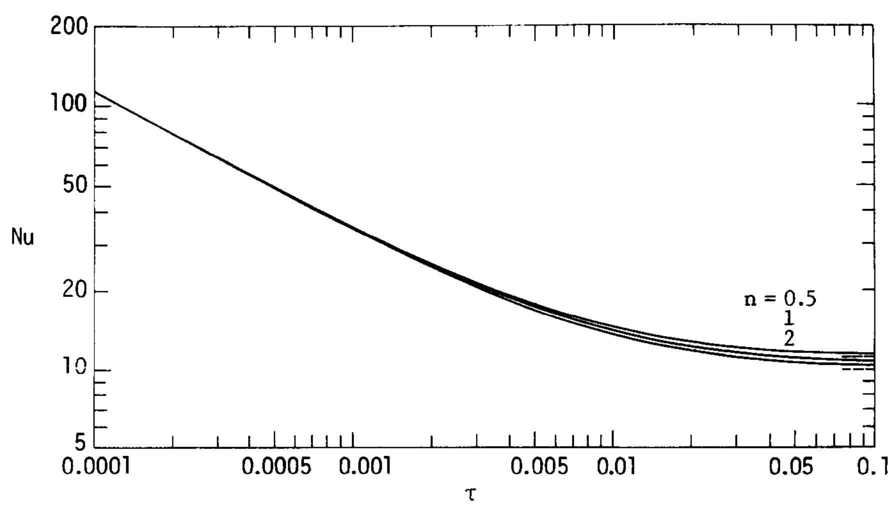


FIG. 5. Effect of flow index on transient heat transfer of pipe, $Br = 0$, $Gz = 1000$. ---designates steady-state value.

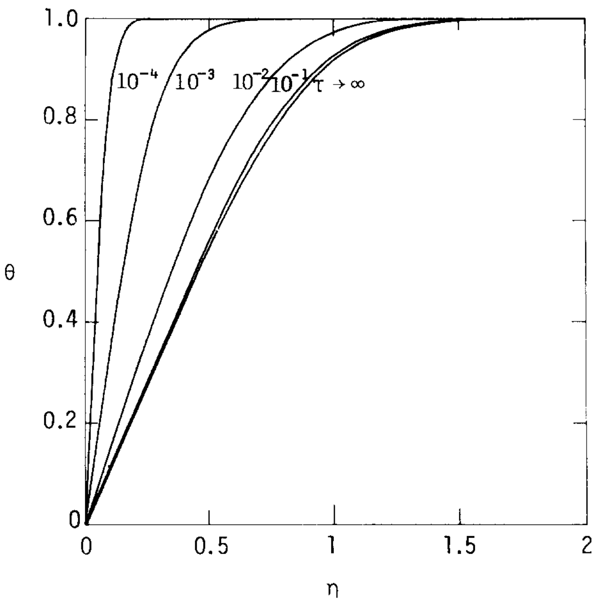


FIG. 6. Development of temperature profile of pipe, $n = 0.5$, $Br = 0$, $Gz = 1000$.

where the last term of (32) represents the viscous dissipation. The initial and boundary conditions are assumed as

$$T(x, y, 0) = T_0 \tag{33}$$

$$T(0, y, t) = T_0 \tag{34}$$

$$\frac{\partial T}{\partial y}(x, 0, t) = 0 \tag{35}$$

$$T(x, H, t) = T_0 + (T_s - T_0)\,1(t). \tag{36}$$

Introducing the dimensionless groups and variables

$$Y = \frac{y}{H}$$

$$X = \frac{\alpha x}{H^2 u_{max}}$$

$$\tau = \frac{\alpha t}{H^2} \tag{37}$$

$$\Theta = \frac{T - T_s}{T_0 - T_s}$$

$$Br = (m+1)^{n+1} \frac{KH^2}{k(T_0 - T_s)} \left(\frac{u_{max}}{H}\right)^{n+1}$$

we have

$$\frac{\partial \Theta}{\partial \tau} + (1 - Y^{m+1}) \frac{\partial \Theta}{\partial X} = \frac{\partial^2 \Theta}{\partial Y^2} + Br\,Y^{m+1} \tag{38}$$

$$\Theta(X, Y, 0) = 1 \tag{39}$$

$$\Theta(0, Y, \tau) = 1 \tag{40}$$

$$\frac{\partial \Theta}{\partial Y}(X, 0, \tau) = 0 \tag{41}$$

$$\Theta(X, 1, \tau) = 0. \tag{42}$$

To facilitate the analysis by boundary layer approach, the coordinate transformation of (14) is used. Equation (38) becomes

$$\frac{\partial^2 \Theta}{\partial \eta^2} + \frac{3}{2}\xi^{-1}[1 - (1 - \xi\eta)^{m+1}]\,\eta \frac{\partial \Theta}{\partial \eta} + Br\xi^2(1 - \xi\eta)^{m+1} - \xi^2 \frac{\partial \Theta}{\partial \tau} = \frac{3}{2}[1 - (1 - \xi\eta)^{m+1}] \frac{\partial \Theta}{\partial \xi} \tag{43}$$

with boundary conditions given by equations (16)–(18).

The local similarity solution for the steady-state heat transfer is obtained by omitting the term on the right-hand side of (43) and the term containing the partial derivative with respect to time, $\partial\Theta/\partial\tau$. For a wide range of the Graetz number, 100–10 000, the matching of the local similarity solution and the extended Leveque solution is found to be excellent as shown in Fig. 7.

Using the coordinate transformation of (26), equation (43) becomes

$$\frac{\partial^2 \Theta}{\partial \zeta^2} + \left\{\frac{3}{2}\xi^{-3}[1 - (1 - \tau^{1/2}\zeta)^{m+1}]\,\tau\zeta + \frac{1}{2}\zeta\right\} \frac{\partial \Theta}{\partial \zeta} + Br\tau(1 - \tau^{1/2}\zeta)^{m+1} = \tau \frac{\partial \Theta}{\partial \tau} \tag{44}$$

where the term on the right-hand side of (43) has been omitted. The instant-local similarity solution is obtained from equation (44) and the boundary conditions of equations (28) and (29) by deleting the term

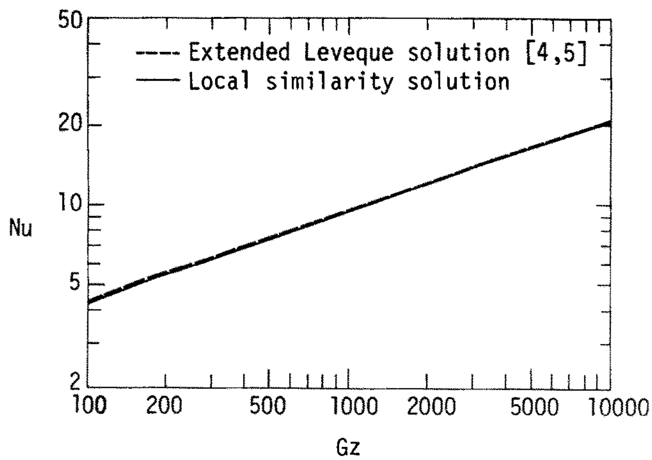


FIG. 7. Comparison of steady-state heat transfer for plate slit flow, $n = 1$, $Br = 0$.

$\tau(\partial\Theta/\partial\tau)$ on the right-hand side of (44) and considering $\tau$ and $\xi$ as prescribed parameters in the numerical integration of the resultant two-point boundary-value problem.

Defining the heat transfer coefficient $h$ as

$$h(T_s - T_0) = k\frac{\partial T}{\partial y}\bigg|_{y=H} \tag{45}$$

the local Nusselt number

$$Nu = \frac{2hH}{k} \tag{46}$$

is then expressed by (23) and (30), respectively, for steady-state and unsteady-state cases. In plate slit case, the Graetz number

$$Gz = \frac{2H}{w}\frac{Q}{\alpha x} \tag{47}$$

where $Q$ is the volumetric flow rate, is related to the transformed axial distance $\xi$ by

$$Gz = 18\left(\frac{m+1}{m+2}\right)\xi^{-3}. \tag{48}$$

The comparison of the local similarity solution with the extended Leveque solution [4, 5] is shown in Fig. 7. The agreement is excellent. Figure 8 shows the effect of flow index on the local Nusselt number. Like the case of pipe flow, the heat transfer rates for large values of normalized time and for steady-state increase as the flow index $n$ decreases.

## 6. CONCLUSION

For small normalized distance it is known that the Graetz problem of the entrance heat transfer in pipes can be converted into boundary layer problem. In this paper, the local similarity method is applied to the analysis of the steady-state Graetz problems for the heat transfer in the entrance region for fully-developed

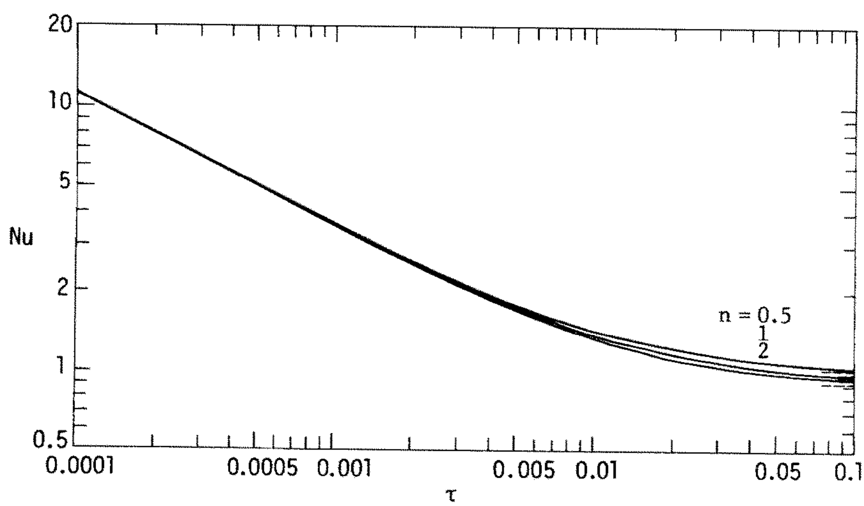


FIG. 8. Effect of flow index on transient heat transfer of plate slit, $Br = 0$, $Gz = 1000$. --- designates steady-state flow.

laminar flow of power-law non-Newtonian fluids in pipes and plate slits. Heat generation from viscous dissipation is taken into account and is represented by the Brinkman number. For the analysis of the unsteady-state Graetz problems, a novel method namely instant-local similarity method is used. The steady-state results agree with the extended Leveque solutions very well, while the unsteady-state heat transfer rates approach the steady-state values asymptotically. Since the normalized time from the transient up to steady state is only of order $10^{-1}$, the instant-local similarity method gives results of high accuracy and is very useful for the study of the unsteady Graetz problems.

*Acknowledgement*—One of the authors (H.-T. L.) acknowledges financial support by the Institute of Nuclear Energy Research, Atomic Energy Council, R.O.C.

### REFERENCES

1. J. Newman, Extension of the Leveque solution, *J. Heat Transfer* **91**, 177–178 (1969).
2. A. McD. Mercer, The growth of the thermal boundary layer at the inlet to a circular tube, *Appl. Sci. Res.* **A9**, 450–456 (1960).
3. Y. P. Shih and J. D. Tsou, Extended Leveque solutions for heat transfer to power law fluids in laminar flow in a pipe, *Chem. Engng J.* **15**, 55–62 (1978).
4. Y. P. Shih and C. C. Chen, Leveque series solutions of laminar annular entrance heat transfer, *J. Chinese Inst. Chem. Engng* **10**, 7–19 (1979).
5. S. M. Richardson, Extended Leveque solutions for flows of power law fluids in pipes and channels, *Int. J. Heat Mass Transfer* **22**, 1417–1423 (1979).
6. S. Y. Tsay and H. S. Yu, Entrance diffusion with chemical reaction in tubular reactors for non-Newtonian flow, *Symposium on Transport Phenomena and Applications*, Taipei, Taiwan, **2**, 147–151 (1980).
7. E. M. Sparrow and R. Siegel, Thermal entrance region of a circular tube under transient heating conditions, *Proceedings of the Third U.S. Congress for Applied Mechanics*, ASME, 817–826 (1958).
8. R. Siegel and E. M. Sparrow, Transient heat transfer for laminar forced convection in the thermal entrance region of flat ducts, *J. Heat Transfer* **81**, 29–36 (1959).
9. R. Siegel, Heat Transfer for laminar flow in ducts with arbitrary time variations in wall temperature, *J. Appl. Mech.* **27**, 241–249 (1960).
10. M. A. Leveque, Les dois de la transmission de la chaleur par convection, *Ann. Mines* **13**, 201–299, 305–362 and 381–415 (1928).
11. E. M. Sparrow and H. S. Yu, Local non-similarity thermal boundary-layer solutions, *J. Heat Transfer* **93**, 328–334 (1971).
12. J. R. Lloyd and E. M. Sparrow, Combined forced and free convection flow on vertical surfaces, *Int. J. Heat Mass Transfers* **13**, 434–438 (1970).
13. H. T. Lin and Y. P. Shih, Laminar boundary layer heat transfer to power-law fluids, *Chem. Engng Comm.* **4**, 557–562 (1980).
14. H. T. Lin and Y. P. Shih, Combined laminar free and forced convection from a vertical plate to power law fluids, *Chem. Engng Comm.* **7**, 327–334 (1980).
15. B. Carnahan, H. A. Luther and J. O. Wilkes, *Applied Numerical Methods*, pp. 405–406. John Wiley, New York (1969).
16. H. T. Lin, Unsteady forced convection heat transfer in laminar boundary layers, Ph.D. Thesis, National Cheng Kung University, Tainan, Taiwan, R.O.C. (1981).

## TRANSFERT THERMIQUE VARIABLE A L'ENTREE DE CONDUITES ET DE FENTES PLANES POUR DES FLUIDES A LOI PUISSANCE

**Résumé**—On propose une méthode de similarité instantanée et locale pour analyser les problèmes de Graetz variables. On étudie le transfert thermique variable pour un écoulement laminaire de fluides non-newtoniens à loi puissance dans la région d'entrée thermiques des conduites et des fentes planes, en considérant la dissipation visqueuse. Seuls sont considérés les grands nombres de Graetz (faible distance axiale normée) et le temps normé jusqu'à la solution permanente est de l'ordre de $10^{-1}$ ; l'approche par la similarité instantanée et locale donne des résultats très précis. Les effets de l'indice de l'écoulement de la dissipation visqueuse et du nombre de Graetz sur le flux de chaleur sont illustrés par des solutions numériques. Les problèmes permanents correspondants sont étudiés par la méthode de similarité locale dont les solutions s'accordent très bien avec les solutions complètes de Lévêque, particulièrement pour des grands nombres de Graetz et de Brinkman.

## INSTATIONÄRER WÄRMEÜBERGANG IM EINLAUFGEBIET VON ROHREN UND PLATTENSPALTEN BEI NICHT-NEWTON'SCHEN FLUIDEN

**Zusammenfassung**—Es wird eine zeitlich-lokale Ähnlichkeitsmethode verwendet, um das instationäre Graetz-Problem zu untersuchen. Behandelt wird der instationäre Wärmeübergang bei voll ausgebildeter laminarer Strömung von viskosen nicht-newton'schen Fluiden im thermischen Einlaufgebeit von Rohren und ebenen Spalten unter Berücksichtigung der viskosen Dissipation. Bei Untersuchungen des instationären thermischen Einlaufs werden nur große Graetz-Zahlen (kleiner normierter axialer Abstand) betrachtet, wobei die normierte Zeit vom Übergangsbereich bis zum stationären Zustand die Größenordnung von $10^{-1}$ hat. Für diesen Bereich liefert die zeitlich-lokale Ähnlichkeitstheorie Ergebnisse von guter Genauigkeit. Die Einflüsse von Strömungsindex, viskoser Dissipation und Graetz-Zahl auf den Wärmeübergang werden anhand numerischer Lösungen gezeigt. Die entsprechenden stationären Graetz-Probleme werden ebenfalls mit der zeitlich-lokalen Ähnlichkeitsmethode behandelt, wobei die Ergebnisse sehr gut mit den erweiterten Leveque-Lösungen, besonders für große Graetz- und Brinkman-Zahlen, übereinstimmen.

## РАСЧЕТ НЕСТАЦИОНАРНОГО ТЕПЛООБМЕНА НА ТЕПЛОВОМ УЧАСТКЕ В ТРУБАХ И ПЛОСКИХ ЩЕЛЯХ ДЛЯ СТЕПЕННЫХ ЖИДКОСТЕЙ

**Аннотация** — Предложен метод локальной автомодельности для решения нестационарных задач Гретца. Исследуется нестационарный теплоперенос при полностью развитом ламинарном течении степенных неньютоновских жидкостей на нагреваемом начальном участке труб и плоских щелей с учетом вязкой диссипации. Решение нестационарных задач теплопереноса рассматривается только для больших значений числа Гретца (небольших нормированных расстояний от входа) и нормированных промежутков времени от переходного до стационарного состояния порядка $10^{-1}$. При таких условиях метод локальной автомодельности дает очень точные результаты. На примере численных решений показано влияние индекса неньютоновости, вязкой диссипации и числа Гретца на интенсивность теплообмена. Методом локальной автомодельности исследуются соответствующие стационарные задачи Гретца. Результаты решения хорошо согласуются с обобщенными решениями Левека в особенности при небольших числах Гретца и Бринкмана.